Modeling the Stress-Strain Behavior of Confined Concrete Columns

KARIM M. EL-DASH [1] and OSAMA O. EL-MAHDY [2]

Synopsis: In this paper, an analytical stress-strain model of confined concrete columns is developed and presented. The model is based on the extensively obtained data from tests of column specimens subjected to concentric compression loading. The tests included a wide range of varieties including both normal and high-strength concretes. The cross sections of the columns were of circular, rectangular, or elliptical shapes. The model incorporates the effective relevant parameters of confinement that have been observed to play important roles in confined column behavior like concrete strength, yield strength of transverse reinforcement, spacing between lateral confining element, and dimensional configuration of column specimen and its transverse reinforcement. The model can be used for concrete confined by spirals, rectilinear hoops, crossties, and combinations of these reinforcements.

The model demonstrates good predictive capability for concrete columns of compressive strength ranging from 20 MPa to 120 MPa. In addition, the model is shown to be applicable for a wide range of quantity and configuration of lateral reinforcement with volumetric ratio to concrete from 0.2% to 4%. The proposed model was compared with the existing experimental results. The comparison showed that the predicted stress-strain relationship obtained using the proposed model provides fine agreement with experimental results with respect to all considered parameters.

Keywords: columns; concrete; confined; ductility; high-strength; model; stress-strain.

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INTRODUCTION

Behavior of confined concrete members has been studied extensively in the last two decades. The effect of lateral reinforcement is not considered up to 40-50% of the concrete maximum capacity, which is the actual working range. The real contribution of confinement takes place at higher range of loading when the lateral strains of concrete become high. The lateral dilation of concrete forces the lateral confining elements to stretch outside producing excessive internal strains and stresses. This behavior from the concrete towards the confining elements attracts passive pressure that enhances the strength and ductility of concrete (Van Mier ,1986, Mander et al., 1988, Muguruma et al., 1990, Karabinis and Kiousis, 1994, Priestley et al., 1994, Cusson and Paultre, 1995, and Mei et al., 2001).

Behavior of concrete under concentric compressive load is governed by bond stresses between the paste and aggregates. When the applied load approaches the ultimate capacity of concrete, slippage between paste and aggregates occurs. This slippage is accompanied by crack initiation that propagates with the incremental increase of loading. If excessive lateral pressure is applied to the concrete, the contact bond will be stronger and the slippage between paste and aggregates will be delayed to a higher range of loading. When the confining reinforcement is sufficient to resume tolerable confining stress, the propagation of cracks will be slower than propagation of cracks for unconfined concrete. The slower rate of propagation causes the better ductility behavior obtained for confined concrete under compressive loading (Van Mier, 1986).

In the current study, it is targeted to establish a comprehensive stress-strain relationship of confined concrete columns subjected to concentric compressive load. The model considers columns confined by spirals or ties with/without cross ties. The presented analytical relationship considers columns made by concrete with compressive strength starting with conventional strength of 20 MPa up to high compressive strength of 124 MPa. The cross sections of the columns employed were of circular, rectangular, or elliptical shapes. The model incorporates the effective relevant parameters of confinement that have been observed to play important roles in confined column behavior like concrete strength, yield strength of transverse reinforcement, spacing between lateral confining element, and dimensional configuration of column specimen and its transverse reinforcement.

The peak strength and peak strain of the stress-strain relationship are presented in a comparative study for different confinement situations. The ascending and descending branches of the stress-strain curve are incorporated in a single equation to produce a single expression for the overall needed relationship. It is targeted in the study to introduce a simple comparative model that predicts, in a high capacity, the anticipated strength and ductility enhancements due to lateral confinement. The model is applicable for concrete columns of compressive strength ranging from 20 MPa to 120 MPa. In addition, the model is shown to be applicable for a wide range of quantity and configuration of lateral reinforcement with volumetric ratio to concrete from 0.2% to 4%.

ANALYTICAL MODEL

In the presented model, a fractional equation is used to predict the stress-strain relationship of laterally confined concrete. This equation has been used by Sargin et al., 1971, Ahmad and Shah, 1982, Martinez et al., 1984, and El-Dash, 1995, to predict the stress-strain curve for different types of columns. The equation is given by;

$$y = \frac{Ax + (B-1)x^2}{1 + (A-2)x + Bx^2} \tag{1}$$

where; $y = f_c / f_{cc}$, $x = \varepsilon_c / \varepsilon_{cc}$, $A = E_c / E_p$,

f_c is the confined concrete stress at strain of ε_c, and f_{cc} and ε_{cc} are the peak stress and the corresponding strain.

The parameter A controls the slope of the ascending branch of the curve depending on the modulus of elasticity, E_c that is calculated as per ACI-318, 2002, equation;

$$E_c = 0.036 w_c^{1.5} \sqrt{f_c^{'}} \tag{2}$$

where w_c is the unit weight of concrete in kg/m^3 and $f_c^{'}$ is in MPa.

The ascending branch is finished at the peak point that provides the model with plastic modulus, E_p, expressed as;

$$E_p = f_{cc} / \varepsilon_{cc} \, . \tag{3}$$

The parameter B, which primarily controls the shape of the stress-strain curve in the post-peak portion, is determined by establishing the strain of one representative point in the post-peak portion of the curve. This point used to be at 85% of the peak stress on the descending branch with the corresponding strain denoted as; ε_{85}. For high-strength concrete, the stress-strain relationship is very sensitive in the post-peak portion. The representative point chosen for strain in the post-peak portion is at 50% of the maximum stress, ε_{50}. The 50% strength post-peak point gives a good representation for the shape of post-peak portion of the curve. It is an intermediate location on the descending portion, far from the sensitivity zone near the peak point, and ahead of the tail end of the curve. This point was utilized by Cusson and Paultre, 1994, in the test measurements and in their analytical model, 1995. In Equation (1), when x >1, the value of y should not be less than 0.2.

To predict the peak stress and the corresponding strain, the following formulations are utilized;

$$f_{cc} = f_{co} + \Delta f_c \quad (4)$$

and

$$\varepsilon_{cc} = \varepsilon_{co} + \Delta \varepsilon_c \quad (5)$$

where; f_{co}, and, ε_{co}, are the peak stress and strain of the unconfined concrete, and, Δf_c, and, $\Delta \varepsilon_c$, are the enhancements in concrete strength and the corresponding strain due to lateral confinement, respectively.

The parameters primarily influencing the stress-strain relationship of confined concrete include the strength of concrete, yield strength of the confining reinforcement, volumetric ratio of the confining reinforcement to the concrete core as well as spacing between confining reinforcement, dimensions of the column, and the configuration of the lateral confining reinforcement. All these parameters are considered in the presented model. Table (1) presents the experimental work used in the analysis including 157 concrete specimens with different cross sections, heights, compressive strength, and transverse reinforcements.

Lateral Pressure

The strength capacity of concrete columns varies considerably with the amount and spacing of lateral confining reinforcement, and the strength of unconfined concrete. The lateral confining pressure in the case of confined circular columns can be easily quantified because the lateral pressure is almost uniform. For rectangular or elliptical column cross-sections, the distribution of the lateral confining pressure is not uniform. The configuration of the transverse reinforcement plays a big role in the behavior of such columns. Besides the variability in the confining pressure due to the shape of the cross section of the column, there is a variability of the pressure in the longitudinal direction due to the spacing between the hoops or the pitch of the spiral.

It is common to assume that when the concrete reaches its maximum resistance, the confining pressure can be computed by assuming that the lateral confining reinforcement yields as it was assumed by Cusson and Paultre, 1995, Hoshikuma et al., 1997, and Razvi and Saatcioglu, 1999-a. Saatcioglu and Razvi, 1998, validated this assumption experimentally for the heavily confined high-strength concrete specimens with volumetric transverse reinforcement ratio of 1.3% for circular cross sections and 2% for rectangular cross sections. For lightly confined columns, the lateral reinforcement may not reach the yield strength but the assumption resulted in acceptable analytical results. In both cases, the consideration of the ratio of the concrete unconfined strength to the yield strength of lateral reinforcement needs to be included in the mathematical expression. A fine measurement for the effective lateral pressure at the maximum resistance can be calculated exploiting the following equation;

$$f_l = k_s\, k_f\, \rho_{st}\, f_{yt} \qquad (6)$$

where; ρ_{st} , is the volumetric ratio of the transverse reinforcement to the confined concrete core and f_{yt} is the yield strength of the transverse reinforcement. Figure (1) shows the relationship between the lateral reinforcement ratio, ρ_{st} , and the enhancement in confined concrete strength. The relationship is almost directly linear proportional with steady enhancement of concrete strength as the transverse reinforcement increase.

The coefficient, k_s, is induced to consider the effect of lateral pressure variability in the vertical direction. Figure (2) presents the relationship between spacing between transverse reinforcement to column breadth, *s/b*, and the enhancement in concrete compressive strength, f_{cc} / f_{co}. It is noticed from the figure that when, s / b, is less than 0.3 the enhancement gained could be considerably higher considering all other affecting parameters. It was deduced in the analysis that columns with rectangular cross section are much more sensitive to the spacing of the lateral reinforcement than the columns with circular cross section. Hence, two different mathematical expressions are presented for this coefficient in the model;

$$k_s = \left(1 - \frac{s}{b}\right)^{0.5} \qquad (7)$$

for circular cross sections and

$$k_s = \left(1 - \frac{s}{b}\right)^{2} \qquad (8)$$

for rectangular cross sections.

The coefficient, k_f , accounts for the change in confining pressure with the change in the ratio of unconfined concrete strength to yield strength of the lateral reinforcement. The factor applies the well-known phenomena that the higher concrete strength columns demand higher lateral reinforcement to obtain same properties enhancements. Figure (3) presents this effect on the enhancement in concrete strength, f_{cc} / f_{co}. It is observed clearly that when, f_{co} / f_{yt}, is less than 0.10 the enhancement in concrete strength could be impressive when other effective parameters are constant. The coefficient is calculated employing the following equation for both rectangular and circular cross sections;

$$k_f = 1 - \left(\frac{f_{co}}{f_{yt}}\right)^{0.5} . \qquad (9)$$

Peak Stress

It is shown in the previous sections and in Figures (1), (2), and (3) that peak stress of confined concrete columns depends primarily upon the strength of unconfined concrete, dimensions of confined core, and amount and configuration of the lateral reinforcement. For the computation of peak stress of normal and high-strength concretes, knowledge of unconfined concrete strength, f_{co}, and the effective lateral pressure, f_l, is needed. The response of confined concrete columns to the lateral pressure varies drastically by the change in the cross section. Columns with circular cross sections experience strength enhancement about double that experienced by columns with rectangular cross section when subjected to same lateral confining pressure. This is referred due the irregularity in the distribution of pressure on the cross section of the rectangular columns. Hence, the following relationship are derived after excessive mathematical calibrations for different types of expressions to represent the confined concrete strength in terms of its unconfined strength and the applied lateral confining pressure.

The following relationship is utilized to predict the compressive strength of confined concrete columns;

$$f_{cc} = f_{co} + 3.8\ f_l \tag{10}$$

for circular cross sections and

$$f_{cc} = f_{co} + 1.8\ f_l \tag{11}$$

for rectangular cross sections. The results of the elliptical concrete columns included in the analysis show mechanical response to the lateral confinement that is similar to that of the rectangular columns.

Figure (4) shows the relationship between the experimentally recorded results for the confined concrete strength versus the values derived analytically from the proposed model. The correlation between the two sets of values is terrific for most of the specimens. The calculated, R^2, value is 0.973 that displays the fine matching between experimental and analytical results.

Peak Strain

The results obtained experimentally for confined concrete specimens proved that high-strength concrete columns require a considerably higher level of lateral confining pressure to simulate the same ductility enhancements of normal strength concrete columns. The variation in the recorded peak strain values is dramatic from one research to another that can be referred to the mix proportions, age of concrete at testing, additive used in the mix, and type of aggregate utilized in the specimen. These parameters are not included neither in the presented study nor in any previous one because of difficulties of quantification of these influences. Records of Razvi and Saatcioglu, 1999-b, and Saatcioglu and Razvi, 1998, shows considerably lower peak strains than those recorded by, Cusson and Paultre, 1994, Hoshikuma et al., 1997, and Lin et al., 2004, for the same concrete strength, dimensions, and transverse reinforcement quantity and configuration. Results obtained by Liu et al., 2000, showed very high values for the peak strain of the confined columns so that it is excluded from the analytical derivation for the peak strain expression.

Figure (5) shows general relationship between the confining pressure ratio to the unconfined strength, f_l / f_{co}, and the peak strain of the experimented specimens. The figure illustrates the proportionality of the confining pressure with the peak strain value in general but a wide scatter can be easily noticed in the figure due to the reasons mentioned earlier. After considerable trials, the following mathematical relationship is found to best fit the relationship between the strain at peak stress value and the effective lateral pressure, f_l / f_{co}. The relationship is:

$$\varepsilon_{cc} = \varepsilon_{co} + 0.57\left(\frac{f_l}{f_{co}}\right)^3 \tag{12}$$

where the peak strain of the unconfined concrete, ε_{co}, is expressed as per the recommendation of Shah and Ahmad, 1994;

$$\varepsilon_{co} = 0.00165 + 0.0000165 f_c^{'} \ . \tag{13}$$

The peak strain mathematical expression has an, R^2, value of 0.718 with respect to the recorded values for the experimental results.

Descending Branch

The shape of the post-peak portion of the stress-strain curves is primarily governed by the parameter, *B*, in Equation (1). To calculate the value of this parameter, a post-peak point is needed to be established. The point at 85% or 50% of the confined concrete strength in the post-peak portion of the curve can be utilized as this reference point. The 85% strength post-peak point is preferred to be used for low strength concrete and well-confined specimens since the descending portion should not have steep descending curve. On the contrary, columns with high-strength concrete and low to medium confinement are sensitive to the strain beyond the peak point. Hence, the point at 50% strength has a good representation for the post-peak portion of the later type. It has an intermediate location on the descending portion, far from the sensitivity zone near the peak point, and ahead of the collapse of the specimen.

A detailed investigation was carried out to find an appropriate mathematical equation that would represent the strain of concrete at 85% and 50% of the peak strength, ε_{85} and ε_{50}, respectively. The strains of concrete in the post-peak portion of the response, ε_{85} and ε_{50}, are found to be correlated to the strength of confined concrete more than to the strength of unconfined strength. Based on the experimental records, the following expressions are found to be most representative of the test data:

$$\varepsilon_{50} = \varepsilon_{cc} + 0.033\left(\frac{f_l}{f_{cc}}\right)^{0.5} \tag{14}$$

and

$$\varepsilon_{85} = \varepsilon_{cc} + 0.021\left(\frac{f_l}{f_{cc}}\right)^{0.5} \ . \tag{15}$$

The above-mentioned mathematical expressions have, R^2, values of 0.94 and 0.73, respectively, with respect to the experimental results included in the study. Once the reference point, ε_{85} or ε_{50}, is established, the parameter, B, can be obtained by back substitution in equation (1). It shows as;

$$B = \frac{1 - Ax - 2x + 2x^2}{x^2} \qquad (16)$$

where; x, is used as for ε_{85} or ε_{50}.

VERIFICATION OF MODEL

Comparisons between the complete stress-strain curves predicted by the proposed model and the experimental results are shown in Figures (6) and (7). Figure (6) shows the results for a square column of 500 mm side length and 1,000 mm height. The specimen was confined by 13 mm diameter welded hoops spaced at 40 mm intervals. The volumetric ratio of the transverse reinforcement was 2.6%. The unconfined compressive strength of concrete was 24.3 MPa and the yield strength of the hoops was 295 MPa. Figure (7) presents the results for a square column of 235 mm side length and 1,400 mm total height. The specimen was confined by 9.5 mm diameter hooked hoops spaced at 50 mm intervals. The volumetric ratio of the transverse reinforcement was 2.8%. The unconfined compressive strength of concrete was 99.9 MPa and the yield strength of the hoops was 705 MPa.

The figures show the effect of lateral confinement on the behavior of concrete columns with respect to the unconfined concrete strength. The lateral confining pressure for the specimen shown in Figure (6) is 4.63 MPa versus 7.99 MPa for the specimen shown in Figure (7). Despite that the lateral pressure for the first specimen is less than the second one, it experienced higher strength and ductility enhancements because its unconfined strength is much less than the second one. In both cases, the proposed model demonstrated high predictive capacity for different values of unconfined concrete strength, cross sectional dimensions, yield strength of transverse reinforcement, and spacing of hoops.

It could be noticed by comparison for the behavior of normal strength and high strength concrete columns that for comparable specimens, the higher strength concrete specimens have lower deformability and energy absorption and dissipation capacities initially. During the latter part of the displacement excursions, these properties improve rapidly and the total values are comparable to those of lower strength concrete specimens. The same conclusion was reported by Bayrak and Sheikh, 1998.

PRACTICAL APPLICATION

The confinement of concrete columns has a neglected effect on strength and ductility up to 40-50% of the compressive strength. The real benefit from confinement arises when the applicable load is close to the concrete strength or beyond this limit. The later situation does not exist in ordinary working stage of loading but may be born because of seismic load or extraordinary situation of loading.

The parameter, $\rho_{st}\, f_{yt}/\, f_{co}$, was used in ACI-ASCE Committee 441, 1997, as a guidance measure for the confinement. Saatcioglu and Razvi, 1998, proposed a minimum value of 0.18 for the term, $\rho_{st}\, f_{yt}\, /\, f_{co}$, for columns with rectangular cross section. In addition, Razvi and Saatcioglu, 1999-b, proposed a minimum value of 0.09 for the same term for concrete columns with circular cross sections. Based on the extensive study carried out in the research, it was reached that a value of 0.10 for, $\rho_{st}\, f_{yt} / f_{co}$, for circular columns most probably results in 20% strength enhancement and ductility index ($\varepsilon_{50} / \varepsilon_{cc}$) of 3. In addition, a value of 0.15 for the same term with rectangular columns may result in 10% strength enhancement and ductility index of 3. The lately mentioned values are recommended for columns severely exposed to quakes or similar cases.

CONCLUSIONS

A numerical model is presented to predict the stress-strain relationship for normal and high strength concrete columns of rectangular and circular sections confined with spirals, ties, and/or cross ties. The model is based on the experimental results of 157 concrete specimens subjected to different types and amounts of transverse reinforcement and tested under concentric loading. Comparisons are made between the predictions of the model and the available experimental results. It can be concluded from the study that:

- The model demonstrates good predictive capability and is applicable for a wide range of variables that include range of unconfined concrete strength from 20 MPa to 120 MPa and transverse reinforcement ratio from 0.2% to 4.9% by volume.
- The strength enhancement, f_{cc} / f_{co}, of the confined concrete decreases with the increase of concrete strength but the total absorbed energy by the column increases with the same strength and lateral confinement configuration.
- The ductility of the confined columns decreases drastically with the increase of concrete strength for the same confinement pattern. Equation (12) shows the exponential relationship between the peak strain and unconfined concrete strength.
- It is realized that a value of 0.10 for, $\rho_{st}\, f_{yt} / f_{co}$, for circular columns most probably results in 20% strength enhancement and ductility index ($\varepsilon_{50} / \varepsilon_{cc}$) of 3. In addition, a value of 0.15 for the same term with rectangular columns may result in 10% strength enhancement and ductility index of 3. The lately mentioned values are recommended for columns severely exposed to quakes or similar cases.

NOTATIONS

E_c	Modulus of elasticity
E_p	Plastic modulus
E_{des}	Deterioration rate
b	Smaller side of column or diameter
f_c	Concrete stress
f'_c	Specified concrete strength
f_{cc}	Confined concrete strength
f_{co}	Unconfined concrete strength
f_l	Lateral pressure
f_{yt}	Yield strength of lateral reinforcement
k, k_1, and k_2	Constants
s	Spacing between hoops

w_c	Unit weight of concrete
ε_c	Concrete strain
ε_{cc}	Confined concrete strain at peak point
ε_{co}	Unconfined concrete strain at peak point
ε_{50}	Concrete strain at 50% of the confined strength on the post-peak branch
ε_{85}	Concrete strain at 85% of the confined strength on the post-peak branch
ρ_{st}	Volumetric ratio of lateral reinforcement to concrete core

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Table 1 -- Experimental work included in the analysis

Author	Number of specimens	Shape	Cross section (mm)	Height (mm)	Compressive strength (MPa)	Transverse reinforcement %
Liu (2000)	12	Circular	D=250	1600	60- 96	0.58 – 3.18
Razvi (1999-b)	20	Circular	D= 250	1500	60 - 124	0.41 – 3.05
Pessiki (1997)	8	Circular	D=559	2235	37.9 – 84.7	1.32 – 2.61
Hoshikuma (1997)	11 13	Circular Rectangular	D=200-500 D=200-1000	600–1500 600-1000	18.5 – 28.8 23.2 – 24.3	0.19 – 4.66 0.39 – 4.66
Saatcioglu (1998)	24	Square	250	1500	60 - 124	0.99 – 4.59
Cusson (1994)	27	Square	235	1400	52.6 – 115.9	1.40 – 4.80
Lin (2004)	24	Square	300	1400	27.6 – 41.3	0.86 – 2.16
Tan (1999)	18	Elliptical	258 - 644	1000	21.2 – 27.8	0.60 - 1.80

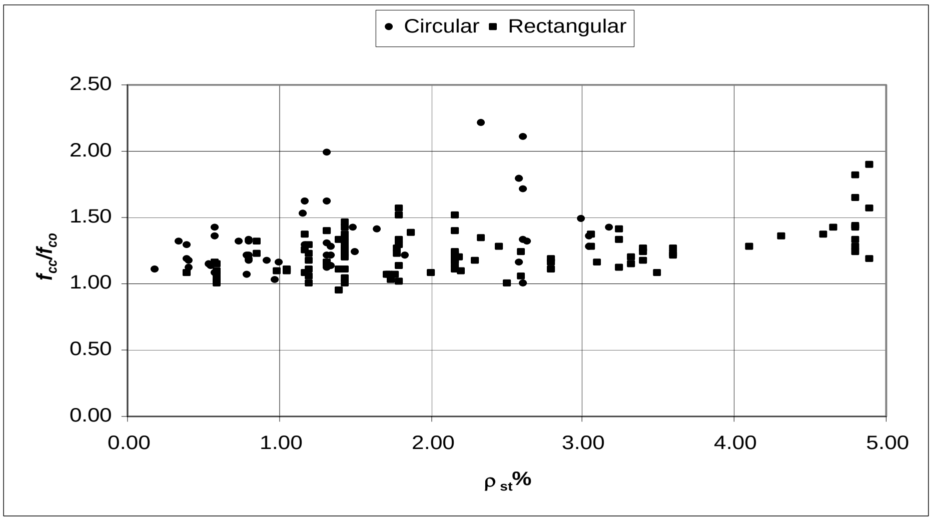

Figure – 1 Relationship between volumetric ratio of lateral reinforcement, ρ_{st}%, and compressive strength enhancement, f_{cc}/f_{co}

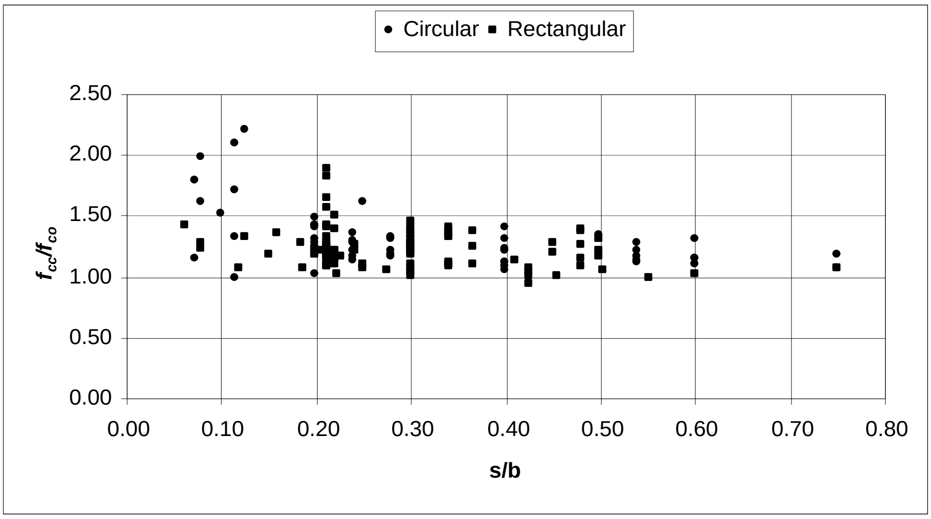

Figure – 2 Relationship between spacing of transeverse reinforcement to column breadth and the compressive strength enhancement

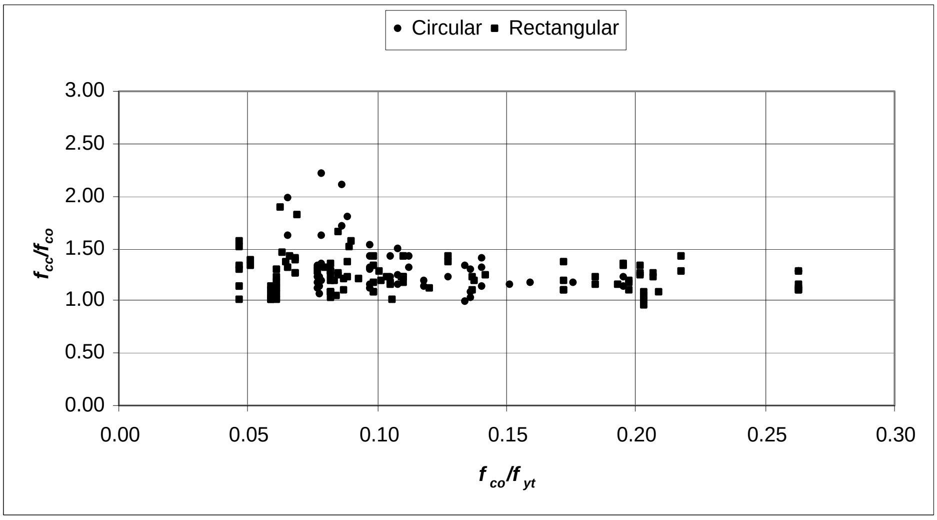

Figure – 3 Relationship between concrete unconfined strength to yield strength of transeverse reinforcement and the compressive strength enhancement

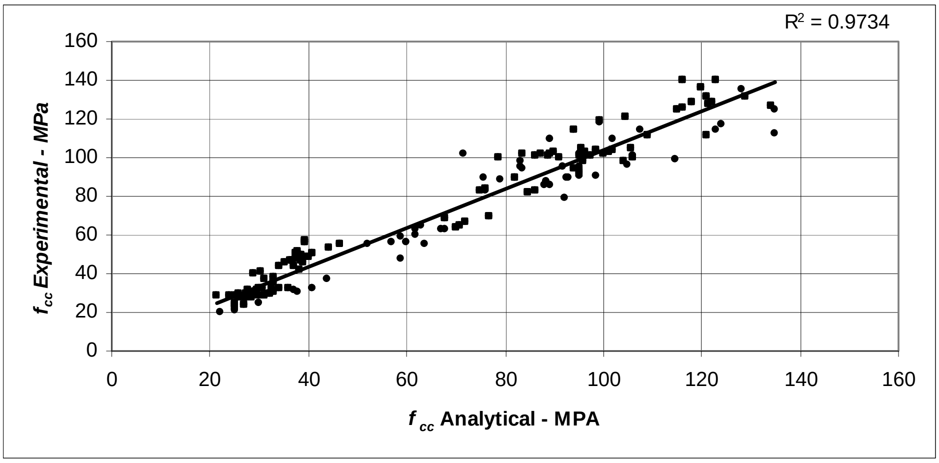

Figure – 4 Correlation between experimental and analytical concrete confined strength

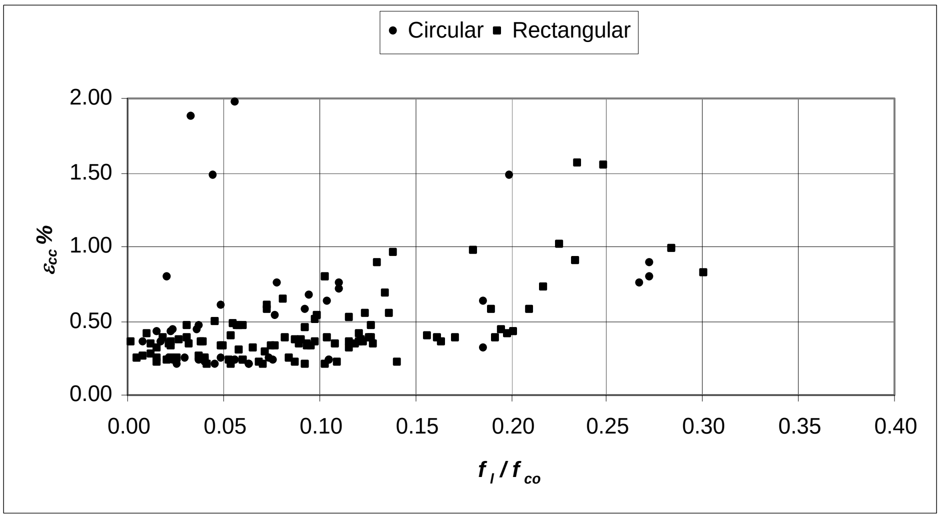

Figure – 5 Relationship between ratio of lateral confining pressure to concrete compressive strength and peak strain

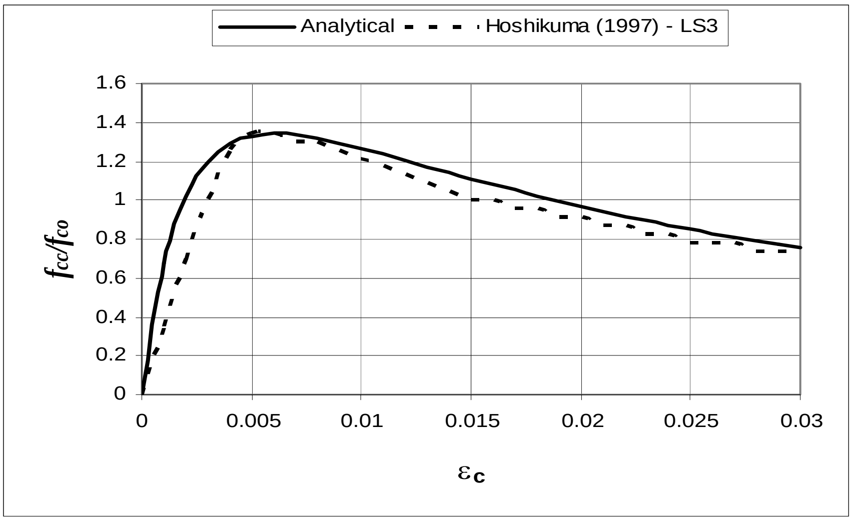

Figure – 6 Analytical vs. experimental stress-strain relationship for LS-3 speciment tested by Hoshikuma et al., 1997

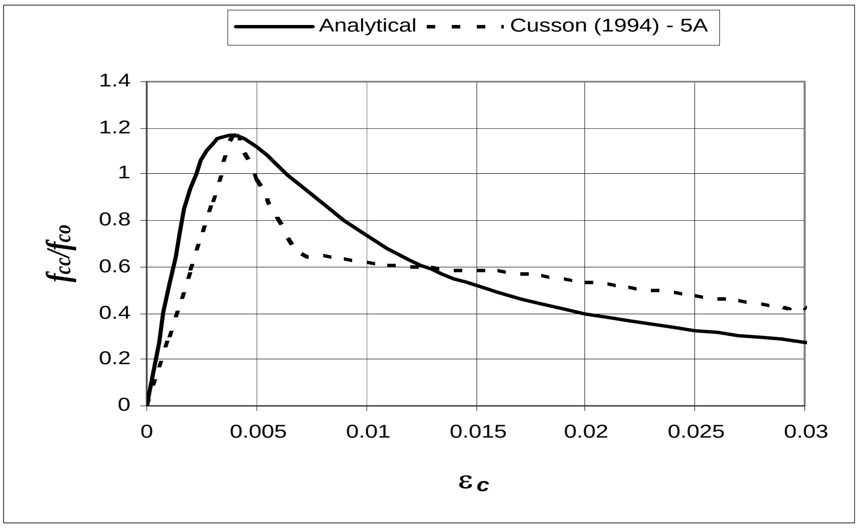

Figure – 7 Analytical vs. experimental stress-strain relationship for A-5 speciment tested by Cusson & Paultre, 1994